

Multiverse in the Third Quantized Horava-Lifshits Theory of Gravity

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Abstract

In this paper we analyze the third quantization of Horava-Lifshits theory of gravity without detail balance. We show that the Wheeler-DeWitt equation for Horava-Lifshits theory of gravity in minisuperspace approximation becomes the equation for time-dependent harmonic oscillator. After interpreting the scaling factor as the time, we are able to derive the third quantized wavefunction for multiverse. We also show in third quantized formalism it is possible that the universe can form from nothing. After we go on to analyze the effect of introducing interactions in the Wheeler-DeWitt equation. We see how this model of interacting universes can be used to explain baryogenesis with violation of baryon number conservation in the multiverse. We also analyze how this model can possibly explain the present value of the cosmological constant. Finally we analyze the possibility of the multiverse being formed from perturbations around a false vacuum and its decay to a true vacuum.

1 Introduction

Wheeler-DeWitt equation is basically the Schroedinger equation for the universe. Just like in first quantized quantum mechanics there is no way to account for the creation and annihilation of particles, there is no way to account for the topology of the universe to change by the creation and annihilation of universes in the second quantized Wheeler-DeWitt equation [1]. However it is well known that we interpret the Schroedinger wave equation as a classical field equation, in the second quantized formalism. We account for the creation and annihilation of particles by modifying the original Schroedinger wave equation by the addition of non-linear terms to it. In analogy with second quantization of the Schroedinger wave equation, in third quantization the Wheeler-DeWitt equation is viewed as a classical field equation generated by a certain classical action. The classical field theory thus obtained is then third quantized. The addition of non-linear terms to the Wheeler-DeWitt equation can account for the topology change generated by the creation and annihilation of universes. Thus third quantization naturally gives rise to a theory of many universes which is called the multiverse [2].

The third quantization has been discussed implicitly in Refs. [3, 4] and explicitly in Refs. [5, 6]. The modification of Wheeler-DeWitt equation by the addition of non-linear terms and the third quantization of the resultant theory was formally analyzed in Ref. [7]. Third quantization of Brans-Dicke theories [8] and Kaluza-Klein theories [9] has also been done. Coherent states for the multiverse have also been studied in the third quantized formalism [10, 11].

The existence of the multiverse was first proposed for giving a consistent explanation to the measurement problem in quantum mechanics and in this context it was called the Everett's many-world interpretation of the quantum theory [12]. This idea also appeared in the landscape of the string theory [13, 14]. In string theory the number of different vacuum states is estimated to be around 10^{500} [15]. It is suspected that all these different vacuum states could be real vacuum states of different universes [16]. The transition from one vacuum state to another has been used as an explanation for inflation in the chaotic inflationary multiverse [17].

In this paper we will study the third quantization of the Horava-Lifshits theory of gravity [20, 21]. Horava-Lifshits theory of gravity is a ultraviolet completion of gravity such that it reproduces general relativity in the infrared limit. Horava-Lifshits theory of gravity is motivated by the fact that the addition of higher order curvature invariants leads to a renormalizable theory of gravity [18], but at the cost of ghost states which spoil the unitarity of the theory [19]. By breaking the Lorentz invariance of the theory and having different Lipschitz scaling for the space and time, it is possible to add higher order spatial derivatives without adding any higher order temporal derivative. Thus it is possible to avoid ghost states but at the cost of breaking Lorentz invariance, in a renormalizable theory of gravity. As the theory uses the concept of Lipschitz scaling from solid state physics, it is generally called Horava-Lifshits theory of gravity.

The original Horava-Lifshits theory of gravity is based on two assumptions which are called the detailed balance and the projectability [22]. The projectability condition is related to the invariance with respect to time reparametrization and therefore to the Wheeler-DeWitt equation. However as both these assumptions of Horava-Lifshits theory of gravity are made just for simplification, it is possible to discuss a more generalized Horava-Lifshits theory of gravity without these assumptions. In fact a generalization to the Horava-Lifshits theory of gravity without detailed balancing has already been done [23, 24]. In this paper we will analyze the Wheeler-DeWitt equation for generalized Horava-Lifshits theory of gravity without detailed balance in third quantized formalism. We will then study multiverse in this theory.

2 Wheeler-DeWitt Equation For Horava-Lifshits Theory of Gravity

For Einstein gravity, the Friedman-Robertson-Walker metric is given by

$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2. \quad (1)$$

where $d\Omega_3^2$ is the usual line element on the three sphere and N is the lapse function and here we have set $k = 1$. In this background, we have $R_{ij} = 2\gamma_{ij}/a^2$

and $R = 6/a^2$. Now following Horava [20], we take the space and time to exhibit the following Lipschitz scale invariance

$$\begin{aligned} t &\rightarrow \ell^3 t, \\ x^i &\rightarrow \ell x^i. \end{aligned} \quad (2)$$

With this Lipschitz scaling the four dimensional diffeomorphism invariance of the theory is explicitly broken and this intern allows us to consider different kinds of kinetic and potential terms. The kinetic term is taken to be quadratic in time derivatives of the metric but the potential term contains high-order space derivatives. Now if \mathcal{L}_K is the Kinetic term and \mathcal{L}_P is the potential term, then the total action for Horava-Lifshits gravity can be written as

$$S = \int dt d^3x (\mathcal{L}_K - \mathcal{L}_P). \quad (3)$$

Here the Kinetic term is given by

$$\mathcal{L}_K = N\sqrt{g}\frac{2}{\kappa^2} (K^{ij}K_{ij} - \lambda K^2), \quad (4)$$

where K_{ij} is the extrinsic curvature, which is defined by

$$K_{ij} = -\frac{1}{2N}\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i, \quad (5)$$

and $K = K^{ij}g_{ij}$ is its trace. There is no contribution from the shift function to the extrinsic curvature in our model as the Friedman-Robertson-Walker metric does not contain any contribution from the shift function. A general potential term without detail balancing can be written as [23]

$$\begin{aligned} \mathcal{L}_P = & N\sqrt{g}[g_0\zeta^6 + g_1\zeta^4 R + g_2\zeta^2 R^2 + g_3\zeta^2 R^{ij}R_{ij} + g_4 R^3 \\ & + g_5 R (R^{ij}R_{ij}) + g_6 R_j^i R_k^j R_i^k + g_7 R \nabla^2 R \\ & + g_8 \nabla_i R_{jk} \nabla^i R^{jk}], \end{aligned} \quad (6)$$

where the couplings g_a are all dimensionless. We can now use projectability to set $N = 1$. Plugging the action for this generalized Horava-Lifshits theory of gravity without detailed balance into the Friedman-Robertson-Walker metric, and following the usual procedure to obtain the Hamiltonian constraint, we get

$$[\pi_a^2 - \omega^2(a)] = 0, \quad (7)$$

where

$$\begin{aligned} \omega^2(a) = & -\frac{(3\lambda-1)}{\kappa^2} 24\pi^4 [-g_0\zeta^6 a^4 + 6\zeta^4 a^2 - 6\zeta^2(6g_2 + g_3) \\ & - 6(36g_4 + 6g_5 + g_6)a^{-2}]. \end{aligned} \quad (8)$$

Now we promote the canonical momentum π_a to an operator, and so we have $\pi_a = -i\partial_a$. Thus the Wheeler-DeWitt equation corresponding to this classical Hamiltonian constraint is given by

$$\left[\frac{\partial^2}{\partial a^2} + \omega^2(a) \right] \phi[a] = 0. \quad (9)$$

In this section we analyzed the Wheeler-DeWitt equation for Horava-Lifshits theory of gravity without detail balancing. In the conventional second quantized formalism this Wheeler-DeWitt equation would be interpreted as the Schroedinger equation for our universe. However we will see in the next section that in order to study multiverse, it is best to interpret this equation as a classical field equation and then third quantize it.

3 Third Quantization

Wheeler-DeWitt equation when interpreted as a quantum mechanical Schroedinger equation can describe the quantum state of a single universe. However, just like the first quantized Schroedinger equation is not compatible with creation and annihilation of particles, the second-quantized Wheeler-DeWitt equation is not compatible with topological changes associated with the creation or annihilation of baby universes. These topology changing processes can be incorporated naturally by using a third quantized formalism in analogy to use of second quantized formalism to account for the creation and annihilation of particles. Thus third quantization is an ideal formalism to study the multiverse.

We interpret Eq. (9), as a classical field equation of a classical field $\phi(a)$, whose classical action is given by

$$\mathcal{S}[\phi(a)] = \frac{1}{2} \int da \left(\left(\frac{\partial \phi}{\partial a} \right)^2 - \omega^2(a) \phi^2 \right). \quad (10)$$

Now obviously the variation of the third quantized action $\mathcal{S}[\phi(a)]$ given by Eq. (10), will lead to the Wheeler-DeWitt equation (9). If we interpret the scaling factor a as the time variable [25] then we can calculate the momentum conjugate to $\phi(a)$ as

$$p_\phi = \frac{\delta \mathcal{S}[\phi(a)]}{\delta \dot{\phi}} = \dot{\phi}, \quad (11)$$

where

$$\dot{\phi} = \frac{\partial \phi}{\partial a}. \quad (12)$$

This the third quantized Hamiltonian obtained by the Legendre transformation, can be written as

$$\mathcal{H} = \frac{1}{2} p_\phi^2 + \frac{\omega^2(a)}{2} \phi^2. \quad (13)$$

This Hamiltonian given by Eq. (13) is the Hamiltonian for the harmonic oscillator with time-dependent frequency $\omega(a)$. The solution to the classical equation of motion for this oscillator is given by

$$\phi(a) = \rho(a) [c_1 \cos \gamma(a) + c_2 \sin \gamma(a)], \quad (14)$$

where the constants c_1 and c_2 are determined by the boundary conditions used. If we use the boundary condition $\phi_1(a_1) = \phi_1$ and $\phi_2(a_2) = \phi_2$, then we have

$$\begin{aligned} c_1 &= \frac{1}{\sin(\gamma_2 - \gamma_1)} \left(\frac{\phi_1}{\rho_1} \sin \gamma_2 - \frac{\phi_2}{\rho_2} \sin \gamma_1 \right), \\ c_2 &= \frac{1}{\sin(\gamma_2 - \gamma_1)} \left(\frac{\phi_2}{\rho_2} \cos \gamma_1 - \frac{\phi_1}{\rho_1} \cos \gamma_2 \right). \end{aligned} \quad (15)$$

As the scaling factor a is interpreted as the time in this formalism, we can write the third quantized Schroedinger equation for this time-dependent harmonic oscillator as follows,

$$\mathcal{H}\Phi = i\frac{\partial}{\partial a}\Phi. \quad (16)$$

Now we can easily calculate the propagator for this harmonic oscillator with time-dependent frequency,

$$\begin{aligned} G(\phi_2, \phi_1) &= \frac{1}{2^n n!} \left[\frac{1}{2i\pi\rho_1\rho_2 \sin(\gamma_2 - \gamma_1)} \right]^{1/2} \exp \left[\frac{i\phi_2^2}{4} \left(\frac{\dot{\rho}_2}{\rho_2} \right) \right. \\ &\quad \left. - \frac{i\phi_1^2}{4} \left(\frac{\dot{\rho}_1}{\rho_1} \right) \right] \exp \left[-\frac{1}{2} \left(\frac{\phi_2^2}{\rho_2^2} + \frac{\phi_1^2}{\rho_1^2} \right) \right] \sum H_n \left[\frac{\phi_2}{\rho_2} \right] \\ &\quad \times H_n \left[\frac{\phi_1}{\rho_1} \right] \exp \left(-i\gamma \left(n + \frac{1}{2} \right) \right), \end{aligned} \quad (17)$$

where H_n is Hermite polynomials. If $\Phi_n(\phi, a)$ is the amplitude to detect n universes with the scalar factor a in the multiverse, then we have

$$\begin{aligned} \Phi_n(\phi, a) &= \exp[i\alpha_n(a)] \left[\frac{1}{\pi^{1/2} n! 2^n \rho} \right]^{1/2} \exp \left[\frac{i}{2} \left(\frac{\dot{\rho}}{\rho} + \frac{i}{\rho^2} \right) \phi^2 \right] \\ &\quad \times H_n \left[\frac{\phi}{\rho} \right], \end{aligned} \quad (18)$$

where the phase functions $\alpha_n(a)$ are described by

$$\alpha_n(a) = - \left(n + \frac{1}{2} \right) \int_0^a \frac{1}{\rho^2} da'. \quad (19)$$

Thus the wavefunction for the full multiverse is given by

$$\Phi = \sum_n C_n \Phi_n. \quad (20)$$

This wavefunction can be obtained by a unitary transformation from the wavefunction of the usual harmonic oscillator with time-independent frequency [26, 27]. Thus if $\tilde{\Phi}$ is the wavefunction of the time-independent oscillator then Φ can be written as

$$\Phi = \frac{1}{\sqrt{\rho}} U^\dagger \tilde{\Phi}, \quad (21)$$

where $1/\sqrt{\rho}$ is a normalization factor and U is given by

$$U = \exp \left(\frac{i\dot{\rho}\phi^2}{2\rho} \right). \quad (22)$$

Thus in this representation the Hamiltonian \mathcal{H} can be written as

$$\mathcal{H} = U^\dagger \mathcal{H}_0 U. \quad (23)$$

This is the Hamiltonian for Harmonic oscillator with time-independent frequency which is denoted here by ω_0 ,

$$\mathcal{H}_0 = \frac{1}{2} p_{0\phi}^2 + \frac{\omega_0^2}{2} \phi_0^2. \quad (24)$$

Now if b_0 and b_0^\dagger are the usual creation and annihilation operators given by

$$\begin{aligned} b_0 &= \sqrt{\frac{\omega_0}{2}} \left(\phi_0 + \frac{i}{\omega_0} \dot{\phi}_0 \right), \\ b_0^\dagger &= \sqrt{\frac{\omega_0}{2}} \left(\phi_0 - \frac{i}{\omega_0} \dot{\phi}_0 \right). \end{aligned} \quad (25)$$

These creation and annihilation operators satisfy

$$\begin{aligned} [b_0, b_0^\dagger] &= 1, & [b_0, b_0] &= 0, \\ [b_0^\dagger, b_0^\dagger] &= 0. \end{aligned} \quad (26)$$

We can now define a vacuum state as a state annihilated by b_0 ,

$$b_0|0\rangle = 0. \quad (27)$$

Here the vacuum state represents a state of nothing, which has no space, time or matter fields in it. The action of b_0^\dagger on the vacuum state creates a universe, i.e., spacetime. However this vacuum is not uniquely defined as this expression for the creation and annihilation operators is valid only for $\omega = \omega_0$. The time-dependent annihilation and creation operators are given by [28],

$$\begin{aligned} b(a) &= \mu(a)b_0 + \nu(a)b_0^\dagger, \\ b^\dagger(a) &= \mu^*(a)b_0^\dagger + \nu^*(a)b_0, \end{aligned} \quad (28)$$

where

$$\mu(a) = \frac{1}{2} \left(\frac{1}{\rho(a)} + \rho(a) - i\dot{\rho}(a) \right), \quad (29)$$

$$\nu(a) = \frac{1}{2} \left(\frac{1}{\rho(a)} - \rho(a) - i\dot{\rho}(a) \right), \quad (30)$$

with, $|\mu|^2 - |\nu|^2 = 1$. Thus if we start from a vacuum $|0\rangle$, the number of universes formed from nothing at time a will be given by

$$N = \langle 0|b^\dagger(a)b(a)|0\rangle = |\nu(a)|^2. \quad (31)$$

Thus there is a finite probability for the formation of universes from nothing. However as we have third quantized the free Wheeler-DeWitt equation these universes formed from nothing will not interact with each other. In the next section we will analyze a system of interacting universes by modifying the Wheeler-DeWitt equation in a non-linear way.

4 Interactions

In the previous sections we have seen that the Wheeler-DeWitt equation is analogous to Schroedinger wave equation, in the sense it represents the quantum state of a single universe. We have also seen that if we third quantize this equation then it represents the quantum state of an ensemble of non-interacting

universes. This is still not enough to account for topology change. To obtain a theory consistent with topology change we need to include interaction terms. So in this section we will analyze the effect of introducing interactions terms in the third quantized action for the Wheeler-DeWitt equation. We modify the third quantized action given by Eq. (10) as

$$\mathcal{S}_T[\phi(a)] = \mathcal{S}[\phi(a)] + \mathcal{S}_I[\phi(a)]. \quad (32)$$

Now if we go to imaginary time $a \rightarrow ia$, then we can write the Euclidean partition function for this action as

$$Z = \int D\phi \exp -\mathcal{S}[\phi]_{ET}, \quad (33)$$

where $\mathcal{S}[\phi]_{ET}$ is the Euclidean version of $\mathcal{S}[\phi]_T$ obtained by letting $a \rightarrow ia$. We can now calculate the n -point functions for any interacting term by the usual methods.

A simple cubic interaction term given by

$$\mathcal{S}_{EI} = \frac{\lambda}{6} \int da \phi^3(a), \quad (34)$$

will generate a three-point function given by

$$G(a_1, a_2, a_3)_E = -\lambda \int da_0 G(a_1, a_0)_E G(a_2, a_0)_E G(a_3, a_0)_E, \quad (35)$$

where $G(a_1, a_0)_E, G(a_2, a_0)_E$ and $G(a_3, a_0)_E$ are the Euclidean Green's functions obtained from \mathcal{S}_E , which is the Euclidean version of \mathcal{S} . This process represents the splitting of a universe U_1 with scaling factor a_1 into two universes, U_2 and U_3 with scalar factors a_2 and a_3 , respectively. Now if we represent matter and gauge fields collectively by χ and include the contribution from χ in our formalism, then ϕ would also depend on χ , $\phi = \phi(a, \chi)$. Thus the three-point function in reality would represent the splitting of the universe U_1 with scaling factor a_1 and matter and gauge field content χ_1 , into two universes U_2 and U_3 with scalar factors a_2, a_3 and their matter and gauge field contents χ_2, χ_3 , respectively. Now if the total number of baryons and anti-baryons in universe U_1 are n_1 and m_1 , respectively and the universe U_1 has formed from nothing without violating the baryon number conservation, then we have

$$Bn_1 - Bm_1 = 0, \quad (36)$$

where Bn_1 and Bn_2 represent the total baryon number of the baryons n_1 and anti-baryons n_2 , respectively. If the total number of baryons and anti-baryons in the universes U_2 and U_3 are n_2, m_2 and n_3, m_3 , respectively, the baryon number conservation implies that

$$Bn_2 + Bn_3 - Bm_2 - Bm_3 = 0. \quad (37)$$

However the baryon number in the universes U_2 or U_3 need not be separately conserved

$$\begin{aligned} Bn_2 - Bm_2 &\neq 0, \\ Bn_3 - Bm_3 &\neq 0. \end{aligned} \quad (38)$$

Thus after splitting of the universe U_1 , the universe U_2 can have more matter than anti-matter if the universe U_3 has more anti-matter than matter. This will not violate the baryon number conservation as the total baryon number of both the universes is still collectively conserved. This can possibly explain the domination of matter over anti-matter in our universe without violating baryon number conservation [29].

In fact we could also study other forms of potential e.g., ϕ^4 interaction term. In this case we would generate a four-point function given by

$$G(a_1, a_2, a_3, a_4)_E = -\lambda \int da_0 G(a_1, a_0)_E G(a_2, a_0)_E G(a_3, a_0)_E G(a_4, a_0)_E. \quad (39)$$

This can represent collision of two universes with scalar factors a_1 and a_2 to form two new universes with scalar factor a_3 and a_4 , respectively. In fact we can view big bang in this model as the collision of two earlier universes to form our present day universe. Again by introducing matter and gauge fields and repeating the above argument we can show that even in this model one universe can have more matter than anti-matter without violating baryon number conservation.

The process given by Eq. (39) can also be interpreted as the splitting of one universe into three distinct regions of spacetime. It may be interesting to note that in the formation of black holes an initial spacetime gives rise to a black hole, a white hole and another distinct region of spacetime. So it seems that third quantized of gravity with ϕ^4 interaction term would naturally lead to the formation of black holes. In a similar way λ^2 processes in the ϕ^3 theory would naturally lead to the formation of wormholes. We could also discuss virtual processes which give rise to the spacetime foam, in the third formalism. Thus ϕ^3 theory would give rise to virtual wormholes [30] and ϕ^4 theory would give rise to the virtual black hole [31], in the spacetime foam. The low-energy effects of these virtual wormholes or virtual black holes in our universe can be given in terms of an effective interaction Lagrangian density given by

$$\mathcal{L}_{eff} = \sum_i \mathcal{L}^i \phi_i, \quad (40)$$

where the index i labels the different wormholes or virtual black holes and \mathcal{L}^i is the insertion operator at the nucleating event. Now the cosmological constant can be shown to vanish by repeating the argument used in Ref. [32]. However in Ref. [32] only virtual wormholes were considered, but in the present third quantized formalism virtual black holes would also have the same low energy effect on the cosmological constant. However as the universes with matter or anti-matter asymmetry form in pairs, they will remain entangled to each other. This may generate an effective value for the the cosmological constant. In fact as time passes the entanglement will reduce and so will the value of the cosmological constant. This might explain the high value of the cosmological constant at the early states of our universe [33] and its low value now [34].

A interesting consequence of modifying the Wheeler-DeWitt equation by adding interactions is that it is possible that the vacuum of the present multiverse is not a true vacuum. This can be seen by considering the potential term constructed out of a general interaction term as follows:

$$V[\phi] = -\frac{\delta \mathcal{S}_{EI}[\phi]}{\delta \phi}. \quad (41)$$

If this $V[\phi]$ is metastable then there is a false vacuum, and if our multiverse is formed by perturbations around this false vacuum then it can always tunnel to a true vacuum. The amplitude for this tunnelling will be given by

$$Z = \int D\phi \exp -\mathcal{S}_T. \quad (42)$$

In semi-classical approximation this decay rate of the false vacuum to a true one will be given by

$$\Gamma = A \exp -B, \quad (43)$$

where B is the coefficient of the third quantized Euclidean action evaluated at the bounce and A is a product of certain factors and a square root of the absolute value of the determinant of the second variation of the third quantized Euclidean action evaluated at the bounce. Thus if our multiverse is formed by perturbations around a false third quantized vacuum it can tunnel to a true third quantized vacuum. In that case all the structure in our multiverse would be destroyed. The possibility of our universe being formed around a false second quantized vacuum has been already discussed in Ref. [35, 36], where it was concluded that if our universe is formed around a false second quantized vacuum, then there is a finite probability for it to tunnel to a true second quantized vacuum. We have applied this idea to the whole multiverse by considering the multiverse to be formed around a false third quantized vacuum state.

5 Conclusion

In this paper we have obtained the wavefunction for the multiverse by third quantization of the Horava-Lifshits theory of gravity without the detailed balance condition and shown that it leads to the creation of universes from nothing. Furthermore, we have shown that baryogenesis without the violation of the conservation of the baryon number occurs due to the third quantization of a modified Wheeler-DeWitt equation. We also discuss a possible solution to the cosmological constant problem and the consequences of a third quantized false vacuum.

The advantage of using third quantization is that it naturally describes the multiverse. Many results that have been discussed for a single universe in the second quantized formalism, can be easily generalized to the full multiverse in the third quantized formalism. Another advantage of using third quantization is that the potential of second quantized theories becomes the frequency for third quantized theories. It is more convenient to deal with frequency than with potentials.

An important assumption made here is that the scalar factor acts like the time variable [25]. This can only be done in minisuperspace models, it is not clear how these results can be generalized to the full superspace and what can act as time in the full superspace. It is possible to write the Wheeler-DeWitt equation in full superspace as a time-independent Schroedinger wave equation with cosmological constant as its eigenvalue [37, 38]. Thus if we are able to write a time-dependent version of the Wheeler-DeWitt equation in analogy to time-dependent Schroedinger wave equation, which would reduce to the conventional

Wheeler-DeWitt equation for states of fixed cosmological constant, then we could obtain time on the full superspace. After which we can apply the methods developed here for minisuperspace to the full superspace.

References

- [1] S. W. Hawking, Phys. Rev. **D46** 603 (1992)
- [2] A. Vilenkin, Phys. Rev. **D50** 2581 (1994)
- [3] A. Strominger, Phys. Rev. Lett. **52** 1733 (1984)
- [4] S. W. Hawking, Phys. Rev. Lett. **B195** 337 (1987)
- [5] C. Teitelboim, Phys. Rev. **D25** 3159 (1982)
- [6] K. Kucha, J. Math. Phys. **22** 2640 (1981)
- [7] S. B. Giddings and A. Strominger, Nucl. Phys. **B321** 481 (1989)
- [8] L. O. Pimentel and C. Mora, Phys. Lett. **A280** 191 (2001)
- [9] Y. Ohkuwa, Int. J. Mod. Phys. **A13** 4091 (1998)
- [10] S. R. Perez, Y. Hassouni, P. F. G. Diaz, Phys. Lett. **B683** 1 (2010)
- [11] S. R. Perez and P. F. G. Diaz, Phys. Rev. **D81** 083529 (2010)
- [12] H. Everett, Rev. Mod. Phys. **29** 454 (1957)
- [13] S. K. Ashok and M. R. Douglas, **JHEP01** 060 (2004)
- [14] P. C. W. Davies, Mod. Phys. Lett. **A19** 727(2004)
- [15] F. Denef and M. R. Douglas, JHEP. **0405** 072 (2004)
- [16] L. M. Houghton and R. Holman, JCAP. **0902** 006 (2009)
- [17] A. Linde, Mod. Phys. Lett. **A1** 81 (1986)
- [18] Stelle K S, Phys. Rev. **D16** 953 (1977)
- [19] S. W. Hawking and T. Hertog, Phys. Rev. **D65** 103515 (2002)
- [20] P. Horava, JHEP, **0903** 020 (2009)
- [21] p. Horava, Phys. Rev. Lett. **102** 161301 (2009)
- [22] P. Horava, Phys. Rev. **D79** 084008 (2009)
- [23] T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. **102** 251601 (2009)
- [24] A. Wang, R. Maartens, Phys. Rev. **D81** 024009 (2010)
- [25] A. Vilenkin, Phys. Rev. **D30** 509 (1984)
- [26] I. A. Pedrosa, Phys. Rev. **D36** 1279 (1987)

- [27] D. G. Vergel and J. S. Villasenor, Ann. Phys. **324** 1360 (2009)
- [28] A. D. Jan- nussis and B. S. Bartzis, Phys. Lett. **A129** 264 (1988)
- [29] A. D. Sakharov, JETP. Lett. **6** 24 (1967)
- [30] S. W. Hawking, Phys. Rev. **D37** 904 (1988)
- [31] S. W. Hawking, Phys. Rev. **D53** 3099 (1996)
- [32] S. Coleman, Nucl. Phys. **B310** 643 (1988)
- [33] A. D. Linde, Phys. Lett. **B108** 389 (1982)
- [34] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. **D9** 373 (2000)
- [35] S. Coleman and F. De Luccia, Phys. Rev. **D21** 3305 (1980)
- [36] Coleman, Phys. Rev. **D15** 2929 (1977)
- [37] R. Garattini, J. Phys. Conf. Ser. **33** 215 (2006)
- [38] R. Garattini, Class. Quant. Grav. 24 **1627** (2007)